Online Appendix: "Taking Away the Guns"

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1 Theoretical Appendix

1.1 Proof of Proposition 1

This proposition depends on the following Lemmas.

Lemma 1. *It is always more costly to remain neutral than to cooperate with one of the combatants.*

Proof. Let $\kappa(i)$ denote the expected costs associated with membership in group $i \in \{G, R, C\}$, with $\kappa(G) = \rho_R\theta_R, \kappa(R) = \rho_G\theta_G$, and $\kappa(C) = \rho_R(1-\theta_R) + \rho_G(1-\theta_G)$. The statement $[\kappa(C) < \kappa(G)] \wedge [\kappa(C) < \kappa(R)]$ ("staying neutral is less costly than joining either combatant") is never true for any $\rho_G \in (0, \rho_G^{max}), \rho_R \in (0, \rho_R^{max}), \theta_G \in [0,1], \theta_R \in [0,1]$ and $\theta_G + \theta_R = 1$. The statement $[\kappa(C) < \kappa(G)] \wedge [\kappa(C) > \kappa(R)]$ ("staying neutral is less costly than joining G but more costly than joining R") is true if and only if $[\rho_G < \rho_R] \wedge \left[0 \leq \theta_G < \frac{\rho_R - \rho_G}{2\rho_R - \rho_G}\right]$, and $[\kappa(C) > \kappa(G)] \wedge [\kappa(C) < \kappa(R)]$ ("staying neutral is more costly than joining G but less costly than joining R") is true if and only if $[\rho_G > \rho_R] \wedge \left[\frac{\rho_G}{2\rho_G - \rho_R} < \theta_G \leq 1\right]$. The statement $[\kappa(C) > \kappa(G)] \wedge [\kappa(C) > \kappa(R)]$ ("staying neutral is more costly than joining G or R") is true in all other cases: (1) $[\rho_G > \rho_R] \wedge \left[0 \leq \theta_G < \frac{\rho_G}{2\rho_G - \rho_R}\right]$, (2) $[\rho_G < \rho_R] \wedge \left[\frac{\rho_R - \rho_G}{2\rho_R - \rho_G} < \theta_G \leq 1\right]$. □

Lemma 1 shows that the use of indiscriminate violence in irregular war partially solves the combatants' collective action problem by rendering "free-riding" (i.e. staying neutral) more costly than cooperation (**?**). Because civilians absorb damage from both government and rebel violence, being a neutral civilian will always be strictly costlier than cooperating with the combatants – each of whom only absorbs damage inflicted by one side.

Lemma 2. *There exist three equilibrium solutions to (3-5) in which the outcome of the fighting does not depend on the initial balance of forces: government victory, rebel victory and mutual destruction.*

Proof. Define a *government victory equilibrium* of (3-5) as a fixed point satisfying $\frac{\delta C}{\delta t} = 0, \frac{\delta G}{\delta t} = 0, \frac{\delta R}{\delta t} = 0$, $C_{eq} \in [0,\infty), G_{eq} \in [0,\infty), R_{eq} \in [0,\infty)$ and $\pi_G(\mathbf{s}) = 1, \pi_R(\mathbf{s}) = 0$. These conditions are satisfied at

$$C_{eq} = \frac{\rho_R\theta_R + u}{\mu_G} \tag{13}$$

$$G_{eq} = \frac{k}{\rho_R\theta_R + u} - \frac{\rho_G(1-\theta_G) + \rho_R(1-\theta_R) + u}{\mu_G} \tag{14}$$

$$R_{eq} = 0 \tag{15}$$

This equilibrium exists (i.e. yields non-negative equilibrium group sizes) for all $\rho_G \in (0, \rho_G^{max}), \rho_R \in (0, \rho_R^{max}), \theta_G \in [0,1], \theta_R \in [0,1], k \in (0,\infty), u \in (0,\infty)$, with μ_G, μ_R as defined in (1,2).

Define a *rebel victory equilibrium* of (3-5) as a fixed point satisfying $\frac{\delta C}{\delta t} = 0, \frac{\delta G}{\delta t} = 0, \frac{\delta R}{\delta t} = 0$, $C_{eq} \in [0,\infty), G_{eq} \in [0,\infty), R_{eq} \in [0,\infty)$ and $\pi_G(\mathbf{s}) = 0, \pi_R(\mathbf{s}) = 1$. These conditions are satisfied at

$$C_{eq} = \frac{\rho_G\theta_G + u}{\mu_R} \tag{16}$$

$$G_{eq} = 0 \tag{17}$$

$$R_{eq} = \frac{k}{\rho_G\theta_G + u} - \frac{\rho_G(1-\theta_G) + \rho_R(1-\theta_R) + u}{\mu_R} \tag{18}$$

This equilibrium exists (i.e. yields non-negative equilibrium group sizes) for all $\rho_G \in (0, \rho_G^{max}), \rho_R \in (0, \rho_R^{max}), \theta_G \in [0,1], \theta_R \in [0,1], k \in (0,\infty), u \in (0,\infty)$, with μ_G, μ_R as defined in (1,2).

Define a *mutual destruction equilibrium* of (3-5) as a fixed point satisfying $\frac{\delta C}{\delta t} = 0, \frac{\delta G}{\delta t} = 0, \frac{\delta R}{\delta t} = 0$, $C_{eq} \in [0, \infty), G_{eq} \in [0, \infty), R_{eq} \in [0, \infty)$ and $\pi_G(\mathbf{s}) = 0, \pi_R(\mathbf{s}) = 0$. These conditions are satisfied at

$$C_{eq} = \frac{k}{\rho_G(1-\theta_G) + \rho_R(1-\theta_R) + u} \tag{19}$$

$$G_{eq} = 0 \tag{20}$$

$$R_{eq} = 0 \tag{21}$$

This equilibrium exists (i.e. yields non-negative equilibrium group sizes) for all $\rho_G \in (0, \rho_G^{max}), \rho_R \in (0, \rho_R^{max}), \theta_G \in [0,1], \theta_R \in [0,1], k \in (0, \infty), u \in (0, \infty)$, with μ_G, μ_R as defined in (**??**,**??**). □

Lemma 2 shows that, as the fighting unfolds over time, the system in (**??**-**??**) will converge to one of two equilibria of primary interest – government victory or rebel victory – and a third equilibrium in which both combatant populations converge to zero. I now turn to the main claim of Proposition 1 ("a government victory equilibrium is stable iff. the government's rate of selective violence is greater than that of the rebels").

Proof. The stability of the government monopoly equilibrium in (13-15) can be shown through linearization. Assume $\rho_G \in (0, \rho_G^{max}), \rho_R \in (0, \rho_R^{max}), \theta_G \in [0,1], \theta_R \in [0,1]$, with μ_i as defined in (1,2). To ensure non-negative population values in equilibrium, I impose a lower bound on immigration parameter $k > \frac{(\rho_R\theta_R+u)(\rho_G(1-\theta_G)+\rho_R(1-\theta_R)+u)}{\mu_G}$.

Let $\mathbf{J}$ be the Jacobian of the system in (3-5), evaluated at fixed point (13-15).

$$\mathbf{J} = \begin{pmatrix} -\frac{k\mu_G}{\rho_R\theta_R+u} & -\rho_R\theta_R - u & -\frac{\mu_R(\rho_R\theta_R+u)}{\mu_G} \\ 0 & 0 & \frac{\mu_R(\rho_R\theta_R+u)}{\mu_G} - \rho_G\theta_G - u \\ \frac{k\mu_G - (\rho_R\theta_R+u)(\rho_G(1-\theta_G)+\rho_R(1-\theta_R)+u)}{\rho_R\theta_R+u} & 0 & 0 \end{pmatrix} \tag{22}$$

The determinant and trace of $\mathbf{J}$ are

$$\det(\mathbf{J}) = \frac{(-\rho_R\theta_R - u)\left(\frac{\mu_R(\rho_R\theta_R+u)}{\mu_G} - \rho_G\theta_G - u\right)(k\mu_G - (\rho_R\theta_R + u)(\rho_G(1-\theta_G) + \rho_R(1-\theta_R) + u))}{\rho_R\theta_R + u} \tag{23}$$

$$\mathrm{tr}(\mathbf{J}) = -\frac{k\mu_G}{\rho_R\theta_R + u} \tag{24}$$

The equilibrium point (13-15) is stable if all the eigenvalues of $\mathbf{J}$ have negative real parts, or $\det(\mathbf{J}) > 0, \mathrm{tr}(\mathbf{J}) < 0$. These conditions hold if and only if $\frac{\rho_G\theta_G}{\rho_R\theta_R} > 1$. □

1.2 Proof of Proposition 2

This proposition depends on the following Lemma.

Lemma 3. *There exist three equilibrium solutions to (9-11) in which the outcome of the fighting does not depend on the initial balance of forces: government victory, rebel victory and mutual destruction.*

Proof. Define a *government victory equilibrium* of (9-11) as a fixed point satisfying $\frac{\delta C}{\delta t} = 0, \frac{\delta G}{\delta t} = 0, \frac{\delta R}{\delta t} = 0$, $C_{eq} \in [0, \infty), G_{eq} \in [0, \infty), R_{eq} \in [0, \infty)$ and $\pi_G(\mathbf{s}) = 1, \pi_R(\mathbf{s}) = 0$. These conditions are satisfied at

$$C_{eq} = \frac{(1-h)\rho_R\theta_R + u}{\mu_G^*} \tag{25}$$

$$G_{eq} = \frac{k}{(1-h)\rho_R\theta_R + u} - \frac{\rho_G(1-\theta_G) + (1-h)\rho_R(1-\theta_R) + u}{\mu_G^*} \tag{26}$$

$$R_{eq} = 0 \tag{27}$$

This equilibrium exists (i.e. yields non-negative equilibrium group sizes) for all $\rho_G \in (0, \rho_G^{max}), \rho_R \in (0, \rho_R^{max}), \theta_G \in [0,1], \theta_R \in [0,1], k \in (0,\infty), u \in (0,\infty), h \in (0,1)$, with μ_G^*, μ_R^* as defined in (7,8).

Define a *rebel victory equilibrium* of (9-11) as a fixed point satisfying $\frac{\delta C}{\delta t} = 0, \frac{\delta G}{\delta t} = 0, \frac{\delta R}{\delta t} = 0$, $C_{eq} \in [0,\infty), G_{eq} \in [0,\infty), R_{eq} \in [0,\infty)$ and $\pi_G(\mathbf{s}) = 0, \pi_R(\mathbf{s}) = 1$. These conditions are satisfied at

$$C_{eq} = \frac{\rho_G\theta_G + u}{\mu_R^*} \tag{28}$$

$$G_{eq} = 0 \tag{29}$$

$$R_{eq} = \frac{k}{\rho_G\theta_G + u} - \frac{\rho_G(1-\theta_G) + (1-h)\rho_R(1-\theta_R) + u}{\mu_R^*} \tag{30}$$

This equilibrium exists (i.e. yields non-negative equilibrium group sizes) for all $\rho_G \in (0, \rho_G^{max}), \rho_R \in (0, \rho_R^{max}), \theta_G \in [0,1], \theta_R \in [0,1], k \in (0,\infty), u \in (0,\infty), h \in (0,1)$, with μ_G^*, μ_R^* as defined in (7,8).

Define a *mutual destruction equilibrium* of (9-11) as a fixed point satisfying $\frac{\delta C}{\delta t} = 0, \frac{\delta G}{\delta t} = 0, \frac{\delta R}{\delta t} = 0$, $C_{eq} \in [0,\infty), G_{eq} \in [0,\infty), R_{eq} \in [0,\infty)$ and $\pi_G(\mathbf{s}) = 0, \pi_R(\mathbf{s}) = 0$. These conditions are satisfied at

$$C_{eq} = \frac{k}{\rho_G(1-\theta_G) + (1-h)\rho_R(1-\theta_R) + u} \tag{31}$$

$$G_{eq} = 0 \tag{32}$$

$$R_{eq} = 0 \tag{33}$$

This equilibrium exists (i.e. yields non-negative equilibrium group sizes) for all $\rho_G \in (0, \rho_G^{max}), \rho_R \in (0, \rho_R^{max}), \theta_G \in [0,1], \theta_R \in [0,1], k \in (0,\infty), u \in (0,\infty), h \in (0,1)$, with μ_G^*, μ_R^* as defined in (7,8).

□

I now turn to the main claim of Proposition 2 ("if the government confiscates a sufficiently large share of privately-held arms, a coercive advantage is a sufficient, but not necessary condition for victory").

Proof. The stability of the government monopoly equilibrium in (25-27) can be shown through linearization. Assume $\rho_G \in (0, \rho_G^{max}), \rho_R \in (0, \rho_R^{max}), \theta_G \in [0,1], \theta_R \in [0,1]$, with μ_i^* as defined in (7,8). To ensure non-negative population values in equilibrium, I impose a lower bound on immigration parameter $k > \frac{(1-h)(\rho_R\theta_R+u)(\rho_G(1-\theta_G)+(1-h)\rho_R(1-\theta_R)+u)}{\mu_G^*}$.

Let $\mathbf{J}$ be the Jacobian of the system in (9-11), evaluated at fixed point (25-27).

$$\mathbf{J} = \begin{pmatrix} -\frac{k\mu_G^*}{(1-h)\rho_R\theta_R+u} & -(1-h)\rho_R\theta_R - u & -\frac{\mu_R^*((1-h)\rho_R\theta_R+u)}{\mu_G^*} \\ 0 & 0 & \frac{\mu_R^*((1-h)\rho_R\theta_R+u)}{\mu_G^*} - \rho_G\theta_G - u \\ \frac{k\mu_G^* - ((1-h)\rho_R\theta_R+u)(\rho_G(1-\theta_G)+(1-h)\rho_R(1-\theta_R)+u)}{(1-h)\rho_R\theta_R+u} & 0 & 0 \end{pmatrix} \tag{34}$$

The determinant and trace of $\mathbf{J}$ are

$$\det(\mathbf{J}) = \frac{(-(1-h)\rho_R\theta_R - u)\left(\frac{\mu_R^*((1-h)\rho_R\theta_R+u)}{\mu_G^*} - \rho_G\theta_G - u\right)\left(k\mu_G^* - ((1-h)\rho_R\theta_R + u)\left(\rho_G(1-\theta_G) + (1-h)\rho_R(1-\theta_R) + u\right)\right)}{(1-h)\rho_R\theta_R + u} \tag{35}$$

$$\mathrm{tr}(\mathbf{J}) = -\frac{k\mu_G}{(1-h)\rho_R\theta_R + u} \tag{36}$$

The equilibrium point (25-27) is stable if all the eigenvalues of $\mathbf{J}$ have negative real parts, or $\det(\mathbf{J}) > 0, \mathrm{tr}(\mathbf{J}) < 0$. These conditions hold iff $h > 1 - \frac{\rho_G\theta_G}{\rho_R\theta_R}$. □

Proving Corollary 2.1 is straightforward from this stability condition. If $\underline{h} = 1 - \frac{\rho_G\theta_G}{\rho_R\theta_R}$, then $\frac{dh}{d\theta_G} < 0$, $\frac{dh}{d\theta_R} > 0$ for all $\rho_G \in (0, \rho_G^{max}), \rho_R \in (0, \rho_R^{max}), \theta_G \in [0,1], \theta_R \in [0,1]$.

1.3 Proof of Proposition 3

This proposition depends on a modified system of equations

$$\frac{\delta C}{\delta t} = k - \left((\mu_R^* + \iota_R)R_t + (\mu_G^* + \iota_G)G_t - (1-h)\rho_R(1-\theta_R) - \rho_G(1-\theta_G) - u\right)C_t \tag{37}$$

$$\frac{\delta G}{\delta t} = ((\mu_G^* + \iota_G)C_t - (1-h)\rho_R\theta_R - u)G_t \tag{38}$$

$$\frac{\delta R}{\delta t} = ((\mu_R^* + \iota_R)C_t - \rho_G\theta_G - u)R_t \tag{39}$$

and the following Lemma

Lemma 4. *There exist two equilibrium solutions to (37-39) in which the outcome of the fighting does not depend on the initial balance of forces: government victory and rebel victory.*

Proof. Define a *government victory equilibrium* of (37-39) as a fixed point satisfying $\frac{\delta C}{\delta t} = 0$, $\frac{\delta G}{\delta t} = 0$, $\frac{\delta R}{\delta t} = 0$, $C_{eq} \in [0,\infty), G_{eq} \in [0,\infty), R_{eq} \in [0,\infty)$ and $\pi_G(\mathbf{s}) = 1, \pi_R(\mathbf{s}) = 0$. These conditions are satisfied at

$$C_{eq} = \frac{(1-h)\rho_R\theta_R + u}{\mu_G^* + \iota_G} \tag{40}$$

$$G_{eq} = \frac{k}{(1-h)\rho_R\theta_R + u} - \frac{\rho_G(1-\theta_G) + (1-h)\rho_R(1-\theta_R) + u}{\mu_G^* + \iota_G} \tag{41}$$

$$R_{eq} = 0 \tag{42}$$

This equilibrium exists for all $\rho_G \in (0,\infty), \rho_R \in (0,\infty), \theta_G \in [0,1], \theta_R \in [0,1], \iota_G \in [0,\infty), \iota_R \in [0,\infty), k \in (0,\infty), u \in (0,\infty), h \in (0,1)$, with $\mu_R^* = 1 - \frac{\rho_G\theta_G}{\rho_G + (1-h)\rho_R}, \mu_G^* = 1 - \frac{(1-h)\rho_R\theta_R}{\rho_G+(1-h)\rho_R}$.

Define a *rebel victory equilibrium* of (37-39) as a fixed point satisfying $\frac{\delta C}{\delta t} = 0$, $\frac{\delta G}{\delta t} = 0$, $\frac{\delta R}{\delta t} = 0$, $C_{eq} \in [0,\infty), G_{eq} \in [0,\infty), R_{eq} \in [0,\infty)$ and $\pi_G(\mathbf{s}) = 0, \pi_R(\mathbf{s}) = 1$. These conditions are satisfied at

$$C_{eq} = \frac{\rho_G\theta_G + u}{\mu_R^* + \iota_R} \tag{43}$$

$$G_{eq} = 0 \tag{44}$$

$$R_{eq} = \frac{k}{\rho_G\theta_G + u} - \frac{\rho_G(1-\theta_G) + (1-h)\rho_R(1-\theta_R) + u}{\mu_R^* + \iota_R} \tag{45}$$

This equilibrium exists for all $\rho_G \in (0,\infty), \rho_R \in (0,\infty), \theta_G \in [0,1], \theta_R \in [0,1], \iota_G \in [0,\infty), \iota_R \in [0,\infty), \alpha_G \in [0,\infty), \alpha_R \in [0,\infty), k \in (0,\infty), u \in (0,\infty), d \in (0,1)$, with $\mu_R^* = 1 - \frac{\rho_G\theta_G}{\rho_G+(1-h)\rho_R}, \mu_G^* = 1 - \frac{(1-h)\rho_R\theta_R}{\rho_G+(1-h)\rho_R}$. □

I now turn to the main proof of Proposition 3.

Proof. Assume $\rho_G \in (0,\infty), \rho_R \in (0,\infty), \theta_G \in [0,1], \theta_R \in [0,1], \iota_G \in [0,\infty), \iota_R \in [0,\infty), h \in (0,1)$, with $\theta_R > \theta_G$. To ensure nonnegative population values, we also impose a lower bound on immigration parameter $k > \frac{(1-h)\rho_R(1-\theta_R)+\rho_G(1-\theta_G)+u)((1-h)\rho_R\theta_R+u)}{\mu_G^*+\iota_G}$. By linearization, a government victory equilibrium is stable if all the eigenvalues of the Jacobian matrix of the system in (37-39), evaluated at fixed point (40-42), have negative real parts, or $\det(\mathbf{J}) > 0, \mathrm{tr}(\mathbf{J}) < 0$. These conditions hold if $h > \max(\underline{h}^*_{(1)}, \underline{h}^*_{(2)})$, where $\underline{h}^*_{(1)} = 1 - \frac{(\mu_G^*+\iota_G)(\rho_G\theta_G+u)}{(\mu_R^*)\rho_R\theta_R} + \frac{u}{\rho_R\theta_R}$, and $\underline{h}^*_{(2)} = 1 - \frac{(\mu_G^*+\iota_G)(\rho_G\theta_G+u)}{(\mu_R^*+\iota_R)\rho_R\theta_R} + \frac{u}{\rho_R\theta_R}$. □

For Corollary 3.1, we substitute $\mu_R^* = 1 - \frac{\rho_G\theta_G}{\rho_G+(1-h)\rho_R}, \mu_G^* = 1 - \frac{(1-h)\rho_R\theta_R}{\rho_G+(1-h)\rho_R}$ into $\underline{h}^*$, yielding

$$\begin{aligned}\underline{h}^*_{(1)} =& \frac{1}{2\theta_R\rho_R^2}\Big(\rho_R\left(\theta_R\left(\rho_G+2\rho_R\right)-\rho_G\theta_G\left(\iota_G+1\right)+u\left(\theta_R-\iota_G\right)\right) \\ &- \sqrt{\rho_R^2\left(\rho_G^2\left(\theta_G\left(\iota_G+1\right)+\theta_R\right)^2+u^2\left(\theta_R-\iota_G\right)^2+2\rho_G u\left(\theta_G\left(\iota_G\left(-\theta_R+\iota_G+1\right)+\theta_R\right)+\theta_R\left(\theta_R+\iota_G\right)\right)\right)}\Big)\end{aligned} \tag{46}$$

$$\begin{aligned}\underline{h}^*_{(2)} =& \frac{1}{2\rho_R^2\theta_R(\iota_R+1)}\Big(\rho_R(\theta_R(\iota_R+1)(\rho_G+2\rho_R)-\rho_G\theta_G(\iota_G+1)+u(\theta_R-\iota_G+\iota_R)) \\ &+ \sqrt{\rho_R^2\left(\rho_G^2(\theta_G(1+\iota_G+\theta_R(1+\iota_R))^2+u^2(\theta_R-\iota_G+\iota_R)^2+2\rho_G u(\theta_G(\theta_R(-\iota_G+2\iota_R+1)+(\iota_G+1)(\iota_G-\iota_R))+\theta_R(\iota_R+1)(\theta_R+\iota_G-\iota_R))\right)}\Big)\end{aligned} \tag{47}$$

and take the partial derivative with respect to ι_G. In both cases, $\frac{d\underline{h}^*}{d\iota_G} < 0$ for all $\rho_G \in (0, \rho_G^{max}), \rho_R \in (0, \rho_R^{max}), \theta_G \in [0,1], \theta_R \in [0,1], \iota_G \in [0,\infty), \iota_R \in [0,\infty), h \in (0,1)$. In addition, $\frac{d\underline{h}^*_{(2)}}{d\iota_R} > 0$

2 Empirical appendix

2.1 Additional matching solutions

To exploit the micro-level variation in the data, I follow existing studies on civil war and aggregate atomic-level events into artificial spatial cells (**???**). Although standard practice in conflict research has been to divide a study region into a regular grid cells of a fixed size (e.g. 50km×50km), I perform the analysis using an ensemble of spatial resolutions, from a minimum of 5km×5km to a maximum of 100km×100km. This approach helps to ensure that my results are not driven by the selection of an arbitrary geographic scale. More importantly, a variable spatial resolution enables inferences about the local and regional impact of disarmament – as well as local vs. regional incentives to use disarmament in the first place. An additional rationale is that administrative boundaries in the Caucasus were frequently changing during this period, sometimes in a manner endogenous to the fighting – such as the deportation of Terek Cossacks and subsequent transfer of their lands to Chechens (**?**). The use of synthetic spatial units helps to minimize the inference challenges associated with these developments.

I take a similar approach with regard to temporal resolution. For each counterinsurgency case in a locality (as defined by cell size), I record the number of rebel attacks observed within a given temporal treatment window before and after the government's operation. The size of this treatment window varies from a minimum of one week to a maximum of six months. This approach permits the evaluation of the immediate and longer-term consequences of disarmament, as well as the immediate and longer-term incentives for its use.

Although most applied research seeks to achieve balance across pre-treatment covariates with a single matching solution, I follow **?** in employing a more extensive search across multiple matching designs, in an effort to simultaneously maximize covariate balance between treatment and comparison groups and the size of the matched sample. Specifically, I begin with a set of four common matching methods – propensity scores (PS),[1] Mahalanobis distance (MD),[2] and genetic matching

[1] PS minimizes the univariate distance between the propensity scores $D_{PS}(X_i, X_j) = |P(T_i = 1|X_i) - P(T_j = 1|X_j)|$ of two observations X_i and X_j, where $P(T_i = 1|X_i)$ is the conditional probability that observation i assigned to treatment, given observed pre-treatment covariates X_i.

[2] MD minimizes the multivariate distance between two observations X_i and X_j using $D_M(X_i, X_j) = \sqrt{(X_i - X_j)\mathbf{S}^{-1}(X_i - X_j)}$ where $\mathbf{S}$ is the sample variance-covariance matrix.

with and without a nested propensity score model (GMPS, GM)[3] – and apply each to the data at various levels of spatial and temporal aggregation. This approach has a dual purpose. First, it enables me to select a matched sample that minimizes selection bias while maximizing statistical leverage. Second, if results are generally consistent across all or most iterations, I can be reasonably confident that the disarmament effect is not an artifact of the underlying assumptions of any one matching technique.

As with the level of aggregation and choice of matching methods, I applied an iterative approach for the selection of an optimal propensity score model. While the preceding enumeration of covariates is theory-driven, a "kitchen sink" regression of disarmament on all of them is not a very efficient way to model selection into treatment: some of the variables may be stronger predictors that others, some may overlap, and others may influence the probability of disarmament in non-linear, or non-additive ways. Since the purpose of a propensity score model is, above all, to predict treatment selection with a high degree of accuracy given a set of observed covariates, misspecification can be highly consequential in subsequent stages of the analysis. To avoid such problems, I ran a logistic regression model of disarmament on over 92,000 combinations of the above covariates. To allow for more complex relationships, I included smoothed functions of variables among the potential candidates.[4] The optimal model was selected as one with the best in-sample predictive accuracy and goodness of fit, and the lowest degree of multicollinearity.[5] Optimal choices varied by level of aggregation, but the average predictive accuracy in models selected for matching was $AUC = 0.968$. This number has the following interpretation: given a randomly selected pair of treated and non-treated observations, the model will assign a higher propensity score to the treated unit with a probability of 0.968.

The full ensemble of matching solutions is shown in Figure 1, with the number of matched pairs on the horizontal axis and level of imbalance on the vertical axis.[6] A total of 1,650 matching solutions is presented, using each of four methods (PS, MD, GM and GMPS, the first two with and without calipers), at eleven spatial scales and 25 temporal scales. Following **?**, I selected an optimal matching solution from the cluster on the lower-left corner of the plot, such that no other solution appears to its left or bottom. This solution was Genetic matching with a nested propensity score model,[7] with data aggregated as a 5km×5km grid and a treatment window of $\Delta t = 15$ weeks. The total number of matched pairs was 238.

[3] GM uses a genetic search algorithm to search over a space of distance metrics, minimizing $D_G(X_i, X_j) = \sqrt{(X_i - X_j)'(\mathbf{S}^{-1/2})'\mathbf{W}\mathbf{S}^{1/2}(X_i - X_j)}$, where $\mathbf{W}$ is a $k \times k$ positive definite weight matrix and $\mathbf{S}^{1/2}$ is the Cholesky decomposition of the sample variance-covariance matrix. GMPS includes a vector of propensity scores $P(T = 1|X)$ among the covariates on which balance is sought (**?**).

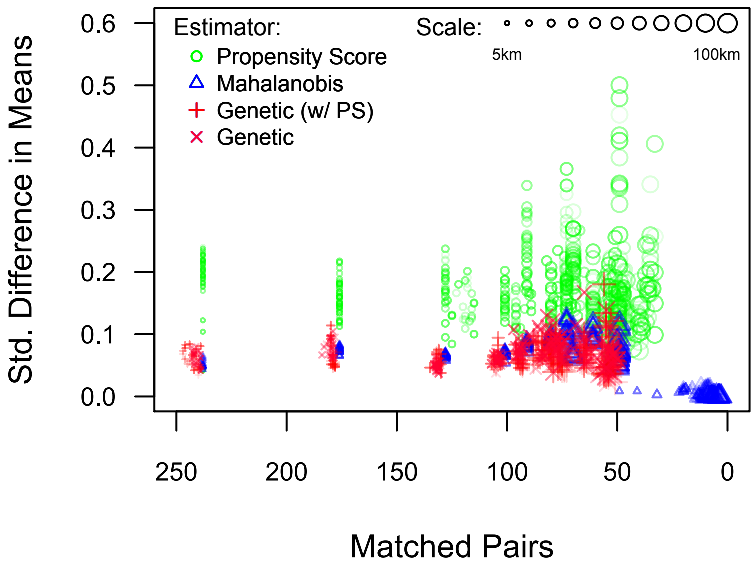
[4] For instance, the geographic coordinates of a locality could enter the model additively (i.e. $LAT_i + LONG_i$) or as a spatial spline (i.e. $f(LAT_i, LONG_i)$).

[5] Predictive accuracy was measured as the area under the receiver-operator characteristic curve (AUC), goodness of fit was measured using the Akaike Information Criterion (AIC), multicollinearity was measured using a combination of maximum pairwise variable correlation and the Variance Inflation Factor (VIF).

[6] The imbalance metric used was average standardized difference in means, or $\text{Imbalance}_m = \frac{1}{K}\sum_k^K \frac{\bar{x}_k^{T(m)} - \bar{x}_k^{C(m)}}{\sigma(x_k^{T(m)})}$, where m indexes the matching solution and k indexes pre-treatment covariates.

[7] The propensity score model used was $\{\text{Disarmament} = \text{logit}^{-1}[\beta_0 + \beta_1\text{Russian} + \beta_2\text{Percent Urban} + \beta_3 log(\text{Population}) + \beta_4\text{Females per 1,000 Males} + \beta_5\text{Border} + \beta_6\text{Prior rebel activity} + \beta_7\text{Prior Disarmament} + \beta_8\text{Year} + f(\text{Month}) + f(\text{Long}, \text{Lat}) + \epsilon]\}$, where $f()$ is a thin-plate spline.

Figure 1: **Ensemble of matching solutions**. Solutions closest to bottom-left corner are optimal. Size of points proportional to scale of geographic aggregation. Transparency proportional to temporal scale (more opaque = larger).



2.2 Additional models

The current section reports several additional results mentioned in the main text. First among these is a model-based estimate of the difference-in-differences reported in the paper's Table 3. The difference-in-differences estimating equation is

$$y_{it} = \beta \text{Post}_t + \delta(T_i \cdot \text{Post}_t) + \alpha_i + \zeta_t + \epsilon_{it} \tag{48}$$

where y_{it} is the number of rebel attacks in unit i during time period t, Post_t is a dummy variable indicating whether time period t is post-treatment, T_i is the treatment indicator, and α_i and ζ_t are vectors of unit and week fixed effects. The difference-in-differences estimate is $\delta = E\left[(Y_{t+\Delta t}(T=1) - Y_{t-\Delta t}(T=1)) - (Y_{t+\Delta t}(T=0) - Y_{t-\Delta t}(T=0))\right]$, where Δt is the size of the post/pre-treatment time window. In the results shown here, I used a windows of 12 weeks.

Table 1 reports the results of the model in 48, estimated on the matched data described above. The estimate is -0.19 (95% CI: -0.267, -0.119) – slightly smaller in magnitude than the non-parametric result reported in the main paper, but still negative and highly significant.

The second set of results, shown in Table 2, is from an additional set of model specifications with alternative distributional assumptions, including Poisson and Negative Binomial, as well as logistic regression with a binary coding of the dependent variable (i.e. some violence vs. no violence). The results shown here are consistent with those in the main text.

Table 1: **Model-based difference-in-differences estimate**

	Dependent variable:
	Rebel attacks
Difference-in-differences (δ)	-0.193^{***}
Std.Err.	(0.038)
95% Confidence Interval	(-0.267, -0.119)
Observations	952 (238 matched pairs $\times$ 2 time periods)
Log Likelihood	-26.706
Akaike Inf. Crit.	573.412
Note:	*p<0.1; **p<0.05; ***p<0.01

Table 2: **Additional models**.

	Dependent variable:		
	Rebel attacks		
	(count)	(count)	(binary)
	GAM Poisson	*GAM Negative Binomial*	*GAM logit*
	(1)	(2)	(3)
Disarmament	−1.223***	−1.836***	−3.410***
	(0.422)	(0.449)	(0.841)
Percent Urban	0.017	0.009	0.026
	(0.029)	(0.030)	(0.039)
log(Population)	2.180**	2.071***	3.122**
	(0.933)	(0.691)	(1.429)
Females/1,000 Males	−0.023*	−0.012*	−0.041
	(0.013)	(0.008)	(0.026)
Russian	1.268**	2.002***	3.967**
	(0.643)	(0.764)	(1.686)
Border	−1.192*	−0.671	−2.379**
	(0.620)	(0.574)	(1.071)
Prior rebel activity	0.181	0.164	0.075
	(0.559)	(0.530)	(0.741)
Prior disarmament	−37.976	−39.819	−36.763
	(6.4e6)	(6.4e6)	(6.4e6)
Year	−0.295	−0.236	−0.677*
	(0.246)	(0.241)	(0.352)
$f(Long, Lat)$	EDF: 9.614	EDF: 4.194	EDF: 11.55
	χ^2: 42.645***	χ^2: 34.46***	χ^2: 37.72***
$f(Month)$	EDF: 1.000	EDF: 1.002	EDF: 1.000
	χ^2: 0.268	χ^2: 0.41	χ^2: 1.287
Constant	560.603	439.813	1,302.918*
	(471.208)	(461.250)	(674.217)
Observations	476	476	476
	(238 matched pairs)	(238 matched pairs)	(238 matched pairs)
AIC	239.340	275.465	136.637
Adjusted R^2	0.501	0.216	0.795
Log Likelihood	−118.277	−136.388	−65.988
UBRE	114.549	110.745	65.038
Note:			*p<0.1; **p<0.05; ***p<0.01

2.3 Cross-national analysis

Although a systematic cross-national analysis of disarmament lies beyond the scope of this article, the current section briefly surveys the relationship between civil conflict and private possession of firearms in modern states.

Because there are currently no comprehensive cross-national data on the timing and location of forcible disarmament campaigns, I focus this inquiry on general state regulations governing private gun ownership. To this end, I merged data on national gun laws from the 2007 Small Arms Survey with data on prior and subsequent intrastate armed conflicts and incidents of political violence.[8] Of 134 countries in the combined dataset, 14 had no state-wide restrictions on private gun ownership apart from age requirements, 116 required either a universal background check or "reason to possess," and 4 had forbidden all civilian possession of firearms.

What kinds of countries are more likely to adopt restrictive gun laws? To answer this question, I ran an ordered logit regression of gun regulations in 2007 (none/background check/forbidden) on countries' prior levels of urbanization, gender balance, inflation, ethno-linguistic fractionalization, civil conflict, rugged terrain and democracy.[9] Table 3 reports these results.

The cross-national predictors of gun control are broadly consistent with those seen in the Caucasus. Consistent with Corollary 2.1, strict gun control policies are less likely where state surveillance and law enforcement capabilities are more robust, such as in relatively urbanized, diverse countries.[10] Consistent with Corollary 3.1, gun policies are more restrictive where poor economic conditions – like a recent history of high inflation – limit the government's ability to co-opt public support.[11]

Do countries with more restrictive gun laws experience less civil conflict? Without cross-national data on when, where and why various gun control policies came into force, this question is difficult to answer beyond a simple statement of correlation. An initial look at cross-national trends since 2007 suggests that the relationship between gun control and civil conflict is negative.[12] Consistent with Proposition 2, countries where civilian gun ownership was forbidden in 2007 experienced a 65 percent lower risk of civil conflict (95% CI: -97.8, 4.6) and 94 percent fewer incidents of political violence (95% CI: -99.5, -71.1) than countries with no restrictions. Although highly tentative, these patterns are consistent with the Soviet case.

[8] Sources include **?** and `GunPolicy.org` for gun regulations, UCDP-PRIO for armed conflicts (**?**) and MEPV for political violence (**?**).

[9] Data sources include **???** and **?**.

[10] According to Model 1 in Table 3, a country with 100 percent urban population prior to 2007 was 93 percent less likely (95% CI: -96.5, -93.1) to outlaw civilian gun ownership than one with 8.8 percent urbanization (lowest in the dataset). A country with an ELF language score of 0.92 was 97.8 percent less likely (95% CI: 96.8, 99.8) to forbid civilian gun ownership that one with an score of 0.002.

[11] A country with average annual inflation of 100 percent in the ten years prior to 2007 was 75.9 percent less likely (95% CI: -100, -3.2) to have no restrictions on civilian gun ownership than a country with 0 inflation.

[12] Model 2 in Table 3 reports a logit regression of civil conflict on gun ownership regulations, and Model 4 is a quasipoisson regression of major political violence incidents.

Table 3: Cross-national regressions.

	Dependent variable:			
	Gun control (2007) *ordered logit*	Civil war (UCDP, post-2007) *logit*	Gun control (2007) *ordered logit*	Civil violence (MEPV, post-2007) *quasipoisson*
	(1)	(2)	(3)	(4)
Gun control (2007)		−2.614*		−1.630***
		(1.411)		(0.523)
Previous civil war	0.321	24.526		
	(0.658)	(1,784.668)		
Previous civil violence			0.002	0.027***
			(0.011)	(0.004)
Urbanization	−0.035**	0.015	−0.036**	−0.002
	(0.016)	(0.018)	(0.016)	(0.012)
Female population	−0.191	−0.039	−0.184	0.012
	(0.180)	(0.382)	(0.179)	(0.133)
Inflation	0.032*	0.007	0.032*	0.003
	(0.018)	(0.009)	(0.018)	(0.011)
Ethnolinguistic	−6.162***	0.408	−6.150***	−0.724
fractionalization	(1.827)	(1.449)	(1.838)	(1.008)
Rugged terrain	0.214	2.827*	0.228	−0.567
	(1.373)	(1.683)	(1.371)	(0.849)
Polity2 score	0.042	−0.245**	0.040	−0.012
	(0.074)	(0.111)	(0.074)	(0.030)
Intercept	-17.276		-17.048	
(none|intermediate)	(9.899)’		(9.814)’	
Intercept	-8.934		-8.728	
(intermediate|forbidden)	(9.542)		(9.462)	
Observations	134	134	134	134
Akaike Inf. Crit.	112.92	66.54	113.14	92.25
Note:				*p<0.1; **p<0.05; ***p<0.01